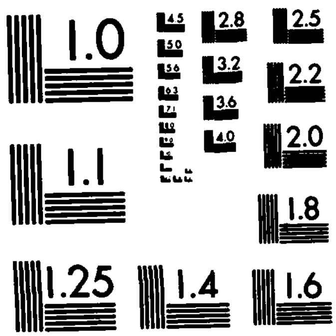

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Final Report on "Confidence Intervals for Functions of Variance Components." Contract Number N00014-78-C-0463

by

Franklin A. Graybill

In this research project confidence intervals were determined for the functions of variance components for the situations listed below. These included one-sided and two-sided confidence intervals.

Since no method is available for exact (exact confidence coefficients) for confidence intervals on the functions of variance components considered, confidence intervals with approximate confidence coefficients were examined. In each case large simulation studies were conducted to determine how good the confidence intervals are. In cases where alternative methods are available they were compared to the procedures in this project.

Confidence intervals were examined for the cases listed below.

Model

1. Two-fold nested model

$Y_{ij} = \mu + A_i + B_{ij} + C_{ijk}$

$\mathcal{E}[A_i] = \mathcal{E}[B_{ij}] = \mathcal{E}[C_{ijk}] = 0$

$\mathrm{Var}[A_i] = \sigma^2_A$; var $[B_{ij}] = \sigma^2_B$

$\mathrm{Var}[C_{ijk}] = \sigma^2_C$. All random variables A_i, B_{ij}, C_{ijk} are independent normal. Also $i = 1, \ldots, I$; $j = 1, \ldots, J$; $k = 1, \ldots, K$.

Confidence intervals on the following were determined

$\sigma^2_A + \sigma^2_B + \sigma^2_C = \sigma^2_{Total} = \sigma^2_T$;

σ^2_A/σ^2_T; σ^2_B/σ^2_T; σ^2_C/σ^2_T,

σ^2_A/σ^2_B, $\sigma^2_A/(\sigma^2_A + \sigma^2_B)$

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2. <u>Three-way cross classification model with interactions</u>

$Y_{ijk} = \mu + A_i + B_j + F_{ij} + C_k + G_{ik} + H_{jk} + P_{ijk}$

$\mathcal{E}[A_i] = \mathcal{E}[B_j] = \mathcal{E}[F_{ij}] = \mathcal{E}[C_k] = \mathcal{E}[G_{ik}] = \mathcal{E}[H_{jk}] = \mathcal{E}[P_{ijk}] = 0.$

$\text{Var}[A_i] = \sigma_A^2$; $\text{var}[B_j] = \sigma_B^2$; $\text{var}[F_{ij}] = \sigma_F^2$; $\text{var}[C_k] = \sigma_C^2$;

$\text{Var}[G_{ij}] = \sigma_G^2$; $\text{var}[H_{jk}] = \sigma_H^2$; $\text{var}[P_{ijk}] = \sigma_P^2$.

The random variables A_i, B_j, F_{ij}, C_k, G_{ik}, H_{jk}, P_{ijk} are independent normal. Also $i = 1, \ldots, I$; $j = 1, \ldots, J$; $k = 1, \ldots, K$.

Confidence intervals on σ_A^2, σ_B^2, σ_C^2 were determined.

3. <u>Two-way cross classification model without interaction</u>

$Y_{ij} = \mu + A_i + B_j + E_{ij}$

$\mathcal{E}[A_i] = \mathcal{E}[B_j] = \mathcal{E}[E_{ij}] = 0$; $\text{var}[A_i] = \sigma_A^2$; $\text{var}[B_j] = \sigma_B^2$;

$\text{var}[E_{ij}] = \sigma_E^2$. The random variables A_i, B_j, E_{ij} are independent normal. Also $i = 1, \ldots, I$; $j = 1, \ldots, J$.

Confidence intervals were determined on the following.

$\sigma_T^2 = \sigma_A^2 + \sigma_B^2 + \sigma_E^2$; σ_A^2/σ_T^2; σ_B^2/σ_T^2; σ_E^2/σ_T^2; σ_A^2/σ_B^2; $\sigma_A^2/(\sigma_A^2 + \sigma_B^2)$; $\sigma_B^2/(\sigma_A^2 + \sigma_B^2)$.

4. <u>Two-way cross classification model with interaction</u>

$Y_{ijk} = \mu + A_i + B_j + G_{ij} + E_{ijk}$

$\mathcal{E}[A_i] = \mathcal{E}[B_j] = \mathcal{E}[G_{ij}] = \mathcal{E}[E_{ijk}] = 0$; $\text{var}[A_i] = \sigma_A^2$; $\text{var}[B_j] = \sigma_B^2$; $\text{var}[G_{ij}] = \sigma_G^2$;

$\text{var}[E_{ijk}] = \sigma_E^2$. The random variables A_i, B_j, G_{ij}, E_{ijk} are independent normal. Also $i = 1, \ldots, I$; $j = 1, \ldots, J$; $k = 1, \ldots, K$.

Confidence intervals were determined for the following.

$\sigma_T^2 = \sigma_A^2 + \sigma_B^2 + \sigma_G^2 + \sigma_E^2$; σ_A^2/σ_T^2; σ_B^2/σ_T^2; σ_G^2/σ_T^2; σ_E^2/σ_T^2.

5. Let $U_i = n_i S_i^2/\theta_i$ be independent chi-square random variables with n_i degrees freedom for $i = 1, 2$, where S_i^2 are observable. Confidence intervals procedures were determined for $C_1\theta_1 + C_2\theta_2$ for $C_1 \geq 0$, $C_2 \geq 0$. These procedures were compared with standard procedures developed by Welch and Satterthwaite and found to be significantly better.

6. For the model considered in 5, a procedure was developed to determine "exact" $1 - \alpha$ confidence coefficients on $C_1\theta_1 + C_2\theta_2$ for $C_1 \geq 0$, $C_2 \geq 0$. The word "exact" means that for any $\varepsilon > o$ the confidence coefficient was within ε of the specified confidence coefficient $1 - \alpha$. These confidence intervals were compared with those in 5 and found to be better but not significantly better.

7. Let $U_i = n_i S_i^2/\theta_i$ be independent chi-square random variables with n_i degrees of freedom for $i = 1, 2, 3$ where the S_i^2 are observable. Approximate confidence intervals were determined and evaluated for $C_1\theta_1 + C_2\theta_2 + C_3\theta_3$ for $C_i \geq 0$. They were found to be better than the conventional procedures developed by Welch and Satterthwaite.

8. Let $U_i = n_i S_i^2/\theta_i$ be independent chi-square random variables with n_i degrees of freedom for $i = 1, 2, 3, 4$ where the S_i^2 are observable. Approximate confidence intervals on θ were determined and evaluated where

$$\theta = \frac{C_1\theta_1 + C_2\theta_2}{C_3\theta_3 + C_4\theta_4}$$

where $C_i > 0$. It was found that a procedure using Satterthwaite's method was quite adequate.

9. Let $U_i = n_i S_i^2/\theta_i$ be independent chi-square rnadom variables with n_i degrees of freedom for $i = 1, 2$, where S_i^2 are observable. Approximate confidence intervals were obtained and evaluated for $C_1\theta_1 - C_2\theta_2$ where $C_i \geq 0$. These confidence intervals were quite good.

10. Let $U_i = n_i S_i^2/\theta_i$ be independent chi-square random variables for $i = 1, 2, 3, 4$ where S_i^2 are observable. Approximate confidence intervals were considered for the functions of θ_i listed below. The procedures are in various stages of completion

(a) $\theta_1 + \theta_2 - \theta_3$

(b) $(\theta_1 + \theta_2)/\theta_3$

(c) $\theta_1 + \theta_2 - \theta_3 - \theta_4$

(d) $(\theta_1 - \theta_2)/\theta_3$.

11. One-factor nested model with unequal numbers

$Y_{ij} = \mu + A_i + E_{ij}$

$\mathcal{E}[A_i] = \mathcal{E}[E_{ij}] = 0$; $\mathrm{var}[A_i] = \sigma_A^2$; $\mathrm{var}[E_{ij}] = \sigma_E^2$. The random variables A_i, E_{ij} are independent normal. Also $j = 1, \ldots, n_i$; $i = 1, \ldots, I$.

Approximate confidence intervals were derived for $\sigma_A^2 + \sigma_E^2$. This work is not yet completed.

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